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THE VISCOSITY OF DRY AND HUMID AIR

J. KESTIN* and J. H. WHITELAW†
Brown University, Providence, R.I.

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Abstract—The paper presents the results of new measurements on the viscosity of dry air along seven isotherms in the range 25–250°C and 1 to 140 atm, subject to an uncertainty of $\pm 0{\cdot}4$ per cent. The zero-density viscosity of dry air can be fitted within the experimental error by an interpolation formula based on the Lennard-Jones 6–12 potential. The apparent molecular constants which lead to a good fit differ considerably from those quoted in the literature. The effect of pressure can be accounted for, at least approximately, by adding a cubic polynomial function of density to the zero-density viscosity. This function, known as the excess viscosity $\Delta\mu$, is independent of temperature, certainly in the range 25–100°C.

The paper also presents measurements of the viscosity of mixtures of air and water vapour. These are restricted to a nominal pressure of 1 atm and cover a range of molar fractions from zero to 0·03 at 25°C and zero to unity at 250°C. Measurements have been performed along the six isotherms 25, 50, 75, 150, 200 and 250°C. It is believed that these are the first measurements of this kind.

NOMENCLATURE

a, b, c,	constants in equation (1);
A, B, C,	constants in equation (6);
b_1,	distance between upper surface of the disc and the lower surface of the upper fixed plate [cm];
b_2,	distance between the lower surface of the disc and the upper surface of the lower fixed plate [cm];
d,	thickness of the disc [cm];
D,	separation of fixed plates [cm];
k,	Boltzmann's constant ($1{\cdot}38044 \times 10^{-16}$) [erg/degK];
I,	moment of inertia of the suspension [g cm^2];
m,	mass of the suspension [g];
M,	molecular weight [g/g mol.];
P,	absolute pressure [int. atm.];
R,	radius of the disc [cm];
t,	temperature [°C];
T,	temperature [°K];
x,	mol fraction;
δ,	(μ observed $-$ μ calculated)/μ observed;
Δ,	logarithmic decrement;
Δ_0,	logarithmic decrement in vacuum;
ϵ,	maximum energy of attraction [erg];
μ,	dynamic viscosity [micropoise];
μ_0,	dynamic viscosity at zero density [micropoise];
$\Delta\mu$,	excess viscosity ($\mu - \mu_0$) [micropoise];
τ,	period of oscillation of suspension [sec];
τ_0,	period of oscillation of suspension in vacuum [sec];
ρ,	density [g/cm^3];
σ,	collision diameter [Å];
$\Omega^{(2\cdot 2)*}$,	reduced collision integral: it is a function of $[(T/\epsilon)k]$, the function being tabulated, for example, in reference 12.

1. INTRODUCTION

A STUDY of available publications on the viscosity of humid air revealed that this important property was known only from calculation. For this reason it appeared appropriate to utilize an existing installation and to contribute a set of measurements on humid air. At the same time the opportunity was taken to extend the range over which the viscosity of dry air had been determined. Only I. F. Golubev [1] had performed measurements at combined moderate

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* Professor of Engineering, Brown University, Providence, R.I.

† Visiting Research Associate, Brown University, Providence, R.I., now at Imperial College, London, S.W.7.

pressures and temperatures, the latter, however, being limited to 100°C as compared with a limit of 250°C available to the present authors. In fact, the present work extended from 25°C to 250°C and from 1 atm to 140 atm in the case of dry air. The measurements on humid air were restricted, in essence, to atmospheric pressure, but covered the complete range of composition wherever practicable, i.e. at 150, 200 and 250°C. At 25, 50 and 75°C, the maximum admissable moisture content was limited by the low values of the saturation pressure of the vapour.

2. METHOD

The measurements were performed with the aid of an oscillating disc viscometer originally constructed for work with steam and adequately described in previous publications [2, 3, 4]. The method was a relative one, calibration being performed with respect to nitrogen at 25°C. The details of calibration, the choice of the standard values and the underlying theory of the instrument have also been described previously [5, 6, 7, 8]. The accuracy of the individual determinations is estimated to be ±0·4 per cent. However, since the systematic deviations in the determination of the viscosity of dry as well as of humid air will tend to be similar, the relative viscosity of humid compared with that of dry air should be of the order of the reproducibility of the instrument, i.e. ±0·1 per cent.

The measurements have been performed at the six nominal temperatures of 25, 50, 75, 150, 200 and 250°C in the case of humid air, and the seven nominal temperatures of 25, 50, 75, 100, 150, 200 and 250°C in the case of dry air, the former over a range of pressures from 1 to 7 atm, the latter over a range of pressures from 1 to 140 atm. The measurements of dry air and those of humid air at 25, 50 and 75°C were previously recorded in reference 9 and were all effected with the same experimental set up. This set up was also used to determine the viscosity of humid air at 150°C. The instrument was then dismantled and reassembled prior to further measurements of the viscosity of humid air at 150, 200 and 250°C.

The air was obtained from commercially available cylinders, at a pressure of 2000 lb/in^2, supplied by the Matheson Co. According to the manufacturer the air was bone dry and free from carbon dioxide. Demineralized water was used for all measurements of humid air.

3. CHARACTERISTICS OF THE SUSPENSION SYSTEM

As already stated, the instrument was assembled, aligned and calibrated twice in preparation for the two series of measurements. The characteristics of the two suspension systems are recorded in Table 1, and a suitable calibration curve was obtained for each of them. A standard reproducibility test showed that differences between measurements under identical nominal conditions differed by less than 0·1 per cent.

4. VISCOSITY OF DRY AIR

The results of the determinations of the viscosity of dry air have been collected in Table 2 and plotted in Fig. 1 in the form of isotherms with density as the independent variable. In the figure the measurements from Table 2 have been

Table 1. Characteristics of the suspension system

radius of disc	$R =$ 3·5049	±0·0002 cm
thickness of disc	$d =$ 0·1015	±0·0001 cm
mass of suspension	$m =$ 13·3973	±0·0001 g
moment of inertia of suspension	$I =$ 53·04	±0·01 g/cm^2
upper separation (25°C)	$b_1 =$ 0·3977	±0·001 cm
lower separation (25°C)	$b_2 =$ 0·3886	±0·001 cm
total separation (25°C)	$D =$ 0·7863	±0·0001 cm
natural period (25°C)	$\tau_0 =$ 33·750 sec	(1st series)
	$\tau_0 =$ 33·460 sec	(2nd series)
wire damping (25°C)	$\Delta_0 =$ 0·00005	

Table 2. The viscosity of dry air

Pressure P (Int. atm)	Temperature t (°C)	Natural period τ_0 (sec)	Period τ (sec)	Density ρ (g/cm^3)	Wire damping Δ_0	Decrement Δ	Viscosity μ (micropoise)
139·47	25·10	33·750	35·403	0·16437	0·00005	0·04862	221·38
109·87	24·73		35·151	0·13089		0·04276	210·56
82·93	24·57		34·983	0·09934		0·036756	201·32
55·10	24·54		34·727	0·06603		0·029359	193·65
27·95	24·52		34·367	0·033383		0·020714	188·39
1·03	24·56		33·774	0·0012214		0·015094	184·40
124·84	49·19	33·875	35·354	0·13466	0·00005	0·044578	222·27
100·82	49·18		35·197	0·10969		0·039884	215·55
77·89	49·14		35·012	0·085160		0·034746	209·81
52·10	49·17		34·769	0·057134		0·027894	204·11
25·80	49·19		34·396	0·028308		0·020198	199·65
1·07	49·14		33·878	0·0011718		0·016093	196·05
142·33	74·15	34·000	35·543	0·13994	0·00005	0·046685	234·85
116·27	74·05		35·379	0·11551		0·042121	228·87
85·51	74·01		35·163	0·08586		0·035816	221·81
55·85	74·01		34·893	0·056462		0·028460	215·84
28·56	74·02		34·517	0·028986		0·021158	211·31
1·07	73·97		33·974	0·0010879		0·0170779	207·51
127·36	103·32	34·080	35·570	0·11517	0·00005	0·043110	240·32
102·52	103·23		35·373	0·093785		0·038531	235·51
78·44	102·19		35·200	0·072536		0·033428	230·83
52·65	101·88		34·947	0·049080		0·027252	227·27
25·09	101·42		34·546	0·023555		0·020844	222·94
1·00	101·42		34·098	0·000939		0·0181398	219·62
100·69	151·21	34·280	35·532	0·081002	0·00007	0·037011	254·00
80·89	151·28		35·369	0·065577		0·033062	251·90
59·11	151·36		35·167	0·048292		0·028473	249·75
40·60	151·33		34·947	0·033379		0·024680	246·99
19·78	151·17		34·642	0·016373		0·0212664	242·69
1·00	151·16		34·301	0·0008317		0·0200200	240·77
104·29	196·66	34·500	35·732	0·075388	0·00013	0·036955	273·35
82·45	197·02		35·554	0·060087		0·032933	271·27
62·44	197·19		35·373	0·045844		0·029180	268·84
41·01	197·30		35·116	0·030349		0·025489	265·78
21·41	197·31		34·837	0·015956		0·022911	262·40
1·00	197·28		34·515	0·0007502		0·021842	260·40
102·86	250·82	34·700	35·951	0·066554	0·00024	0·036293	296·50
82·04	250·82		35·768	0·053540		0·032870	294·25
61·56	250·77		35·550	0·040500		0·029594	291·11
40·60	250·78		35·323	0·026942		0·026696	287·64
20·60	250·69		35·057	0·013777		0·024725	284·82
1·00	250·64		34·787	0·0006738		0·024022	282·97

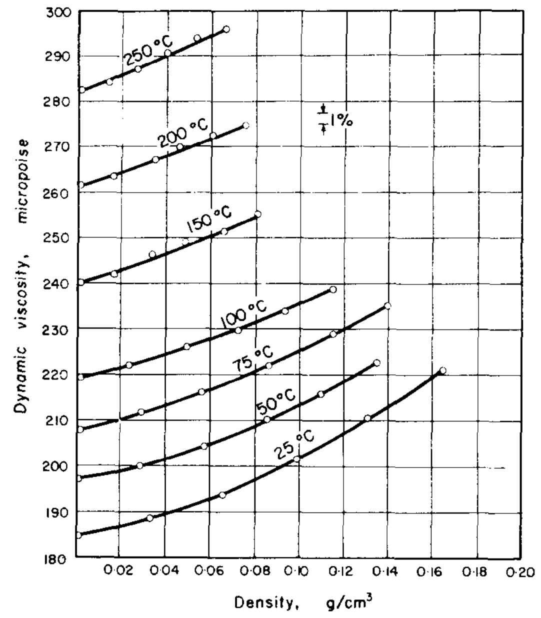

FIG. 1. The viscosity of dry air.

corrected to nominal temperatures and represent true isotherms. The corrections were applied at constant density and were taken to be proportional to the very small residual temperature differences, the effect of density upon them being negligible. This correction contributed at most a change of 0·7 per cent in the viscosity.

All isotherms have been represented by polynomials in density, evaluated by least squares, and it was found that for $T < 150°C$ it was sufficient to employ quadratics, whereas at and above $T = 150°C$ a cubic polynomial expansion

$$\mu = \mu_0 + a\rho + b\rho^2 + c\rho^3 \qquad (1)$$

was needed. The coefficients a, b and c, together with the value of μ_0, were obtained by the method of least-squares and are given in Table 3. The standard deviation did not exceed 0·1 per cent for an isotherm as a whole.

(a) *At zero density.* The zero-density viscosity $\mu_0(T)$ is seen plotted in Fig. 2. It has been compared with the measurements due to M. Trautz and W. Ludewigs [10] and V. Vasilesco [11]. It is seen that the values obtained from the

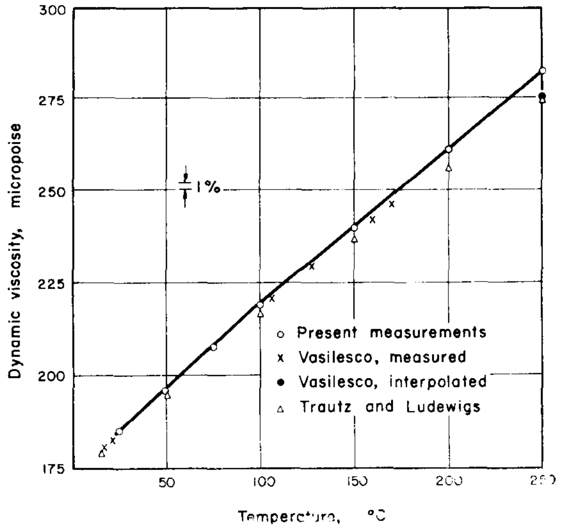

FIG. 2. Zero density viscosity, μ_0, of dry air as a function of temperature.

Table 3. Coefficients of the polynomial expansion

$$\mu = \mu_0 + a\rho + b\rho^2 + c\rho^3$$

Temperature t (°C)	μ_0 (micropoise)	a [micropoise/(g/cm³)]	b [micropoise/(g/cm³)²]	c [micropoise/(g/cm³)³]
25	185·51	91·355	812·28	0
50	196·40	103·55	686·71	0
75	207·80	123·27	528·26	0
100	219·19	120·04	429·60	0
150	239·96	141·4	1880	−19·610
200	261·43	111·1	2918	−27·120
250	282·63	72·2	5070	−46·620

present measurements indicate a somewhat faster rate of increase with temperature. The agreement is excellent at the lowest temperature of 25°C and the discrepancy reaches a value of 3 per cent at the highest temperature of 250°C.

These values were utilized to compute apparent constants for a Lennard-Jones 6–12 potential on the assumption that the viscosity can be interpolated by the standard formula*

$$\mu_0 = \frac{26{\cdot}693\sqrt{(MT)}}{\sigma^2\,\Omega^{(2{\cdot}2)*}}\ \text{micropoise.} \tag{2}$$

Values of the zero-energy separation σ and of the energy minimum ϵ/k have been determined by the use of a digital computer. Employing equation (2) it is possible to determine values of σ and ϵ/k from *pairs* of values of viscosity. Accordingly, values of these two parameters have been computed for the pairs, 25, 250°C; 25, 75°C; 75, 250°C, it being clear that the errors in viscosity can cause very large uncertainties in the values of the apparent molecular constants if the temperature interval is small. In this manner, we obtained

for 25, 250°C $\quad \sigma = 3{\cdot}491$ Å, $\quad \epsilon/k = 123{\cdot}8$°K

for 25, 75°C $\quad \sigma = 3{\cdot}566$ Å, $\quad \epsilon/k = 107{\cdot}2$°K

for 75, 250°C $\quad \sigma = 3{\cdot}466$ Å, $\quad \epsilon/k = 132{\cdot}1$°K

Since it is very time-consuming to arrive at statistically optimum apparent molecular constants to fit the complete range of temperature, we decided to form averages of the above results. Taking into account that the probable errors of measurement increase with temperature and that the uncertainties in the determination of the constants decrease as the temperature range is extended, we arbitrarily assigned weighting factors of 2 to the first two sets of apparent constants.

Hence, we obtained

$$\sigma = 3{\cdot}516\ \text{Å}$$

$$\epsilon/k = 118{\cdot}8\,^{\circ}\text{K} \tag{3}$$

The values of the viscosity of air were then recomputed with the aid of equation (2) and the constants (3). They are quoted in Table 4 together with the fraction

$$\delta = \frac{\mu_{\text{obs}} - \mu_{\text{calc}}}{\mu_{\text{obs}}}$$

It may be seen that the apparent constants (3) together with equation (2) reproduce the measured values to approximately ±0·5 per cent.

As is well known, different sets of values of σ and ϵ/k quoted in the literature (e.g. those listed by J. O. Hirschfelder, C. F. Curtiss and R. B. Bird [12] for a particular gas seldom

Table 4. *The viscosity of dry air at zero density*

	Viscosity		
Temperature t (°C)	Present measured value (taken from Table 3) μ_{obs} (micropoise)	Value calculated from equation (2) and constants (3) μ_{calc} (micropoise)	Deviation $\delta = \frac{\mu_{\text{obs}} - \mu_{\text{calc}}}{\mu_{\text{obs}}}$
25	184·51	184·22	+0·0016
50	196·40	196·38	+0·0001
75	207·80	208·10	−0·0014
100	219·19	219·45	−0·0012
150	239·96	241·10	−0·0048
200	261·43	261·59	−0·0006
250	282·63	281·10	+0·0054

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* This formula represents the viscosity of a pure gas to a first approximation according to the Chapman–Enskog theory of transport phenomena in dilute gases; its derivation can be found in reference 12.

agree among themselves. It is, therefore, impossible to claim that our values represent the true intermolecular constants for air. All that can be said is that the present two values lead to an acceptable interpolation scheme and that they are closest to those which were derived from the second virial coefficient measured by L. Holborn and J. Otto [13].

(b) *At higher density.* In many cases the viscosity excess

$$\Delta\mu = \mu - \mu_0 \tag{4}$$

proves to be a unique function of density, the only term depending on temperature in the expression $\mu = \mu_0 + \Delta\mu$ being the zero-density function $\mu_0(T)$. The question as to whether this is true for our measurements cannot be answered in relation to the coefficients a, b, c in equation (1) and Table 3, which would have to be identical for all isotherms if the above hypothesis were to prove to be correct. This is due to the fact that the coefficients a, b, c merely serve to furnish a usable interpolation formula. Slight changes in a lower-order coefficient are compensated by changes in the higher-order coefficients, and it is not possible to ascribe to the factors a, b, c in equation (1) the physical significance of virial coefficients. For this reason we have prepared the diagram in Fig. 3 which displays a plot of excess viscosity $\Delta\mu$ against density. The diagram shows that the hypothesis that the excess viscosity $\Delta\mu$ is a function of density ρ alone is certainly satisfied for the four isotherms of 25°C, 50°C, 75°C and 100°C. At the higher temperatures there seems to appear a tendency for the viscosity excess $\Delta\mu$ to become sensitive to temperature. However, this trend cannot be accepted as being conclusively present owing to the obvious difficulties which are normally encountered when the temperature level is increased.

If this tendency is ignored, it is possible to represent the average curve in Fig. 3 by the polynomial

$$\Delta\mu = A\rho + B\rho^2 + C\rho^3 \tag{5}$$

with

$$\begin{aligned} A &= 1\cdot112 \times 10^{-4}\ \text{micropoise/(g/cm}^3) \\ B &= 4\cdot248 \times 10^{-4}\ \text{micropoise/(g/cm}^3)^2 \\ C &= 1\cdot566 \times 10^{-3}\ \text{micropoise/(g/cm}^3)^3. \end{aligned} \tag{6}$$

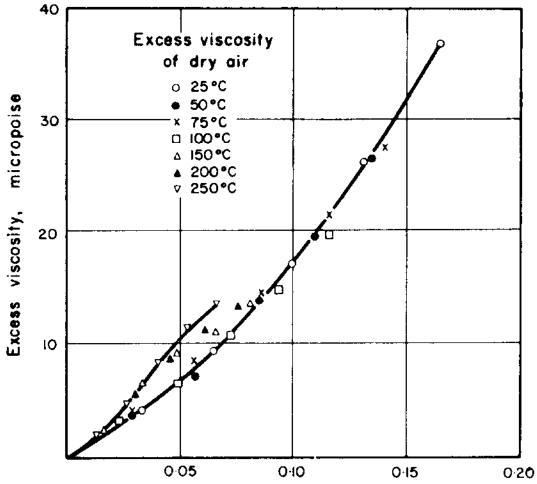

FIG. 3. Excess viscosity in terms of density.

This polynomial fits the data in the range 25–100°C with a standard deviation of 0·5 micropoise. Hence, for many practical applications, it is possible to compute the viscosity of dry air by the combined equation.

$$\frac{\mu}{\text{micropoise}} = \frac{26\cdot963\sqrt{[(28\cdot966)(T/{}^\circ\text{K})]}}{(3\cdot516/\text{Å})\Omega^{(2\cdot2)*}} + 1\cdot112 \times 10^{-4}\left(\frac{\rho}{\text{g/cm}^3}\right) + 4\cdot248 \times 10^{-4}\left(\frac{\rho}{\text{g/cm}^3}\right)^2 + 1\cdot566 \times 10^{-3}\left(\frac{\rho}{\text{g/cm}^3}\right)^3 \tag{7}$$

where $\Omega^{(2\cdot2)*}$ is a function of $(T/118\cdot8\,^\circ\text{K})$.

The experimental results quoted in this paper deviate from equation (7) by maximum amounts which increase with temperature in the following way:

at	25°C	50°C	75°C	100°C
by	0·3%	0·5%	0·5%	0·4%
at	150°C	200°C	250°C	
by	0·7%	1·0%	1·9%	

5. VISCOSITY OF HUMID AIR

The viscosity of humid air was determined by two methods which differed in the details of varying the moisture content in the air. In the range 25–75°C, it was not possible conveniently to exceed a mass of water vapour which corresponds to that filling the vessel at a partial pressure equal to the saturation pressure for a particular isotherm. Above 150°C, it was possible to evaporate controlled quantities of water into the vessel and thereby change the moisture content.

Accordingly, the measurements along the 25, 50 and 75°C isotherms were performed with an essentially constant mass of water vapour and varying mass of air in the instrument. At each temperature the viscometer contained saturated vapour whose partial pressure was fixed by the temperature level. The mass of air was varied by progressively increasing its own partial pressure. Since the partial pressure of water vapour remained constant, it was necessary to increase the total pressure in the instrument.* In this manner, by adding dry air and so increasing the total pressure at constant temperature—in practice from 1 to 7 atm in several steps—the mol fraction of water vapour was reduced. Thus, at a given temperature, the point at the highest mol fraction x_{H_2O} for water vapour was measured at 1 atm total pressure, whereas the point at the lowest value of x_{H_2O} was measured at the highest total pressure, i.e. at 7 atm approx. The value for $x_{H_2O} = 0$ (dry air) was again measured at atmospheric pressure: in practice, the zero-density value was used instead, because the effect of changing the pressure from 0 atm to 1 atm at constant temperature upon the viscosity of air is negligible. The mol fraction x_{H_2O} was calculated from Dalton's law, allowance being made for the deviation of each component from perfect gas behaviour where this proved to be significant.

In order to reduce all measurements to a common pressure of 1 atm, a correction was applied. The measurements were made at elevated pressures, ranging from 1 to 7 atm, which caused the viscosity of each component, μ_{H_2O} and μ_{air}, to have a slightly different value than it would have at the reference pressure at 1 atm. Thus it is necessary to estimate the change in viscosity which would result upon compression from 1 atm to P at constant temperature and composition. Since the correction is expected to be small, a very crude estimate is adequate, accordingly, we based it on C. R. Wilke's equation [14].

$$\mu_{mix} = \frac{\mu_{H_2O}}{1 + \frac{x_{air}}{x_{H_2O}} \cdot \frac{\left[1 + \left(\frac{\mu_{H_2O}}{\mu_{air}}\right)^{1/2} \left(\frac{M_{H_2O}}{M_{air}}\right)^{1/4}\right]^2}{2 \cdot 83 \left[1 + \frac{M_{H_2O}}{M_{air}}\right]^{1/2}}} + \frac{\mu_{air}}{1 + \frac{x_{H_2O}}{x_{air}} \cdot \frac{\left[1 + \left(\frac{\mu_{air}}{\mu_{H_2O}}\right)^{1/2} \left(\frac{M_{air}}{M_{H_2O}}\right)^{1/4}\right]^2}{2 \cdot 83 \left[1 + \frac{M_{air}}{M_{H_2O}}\right]^{1/2}}} \quad (8)$$

Wilke's equation (8) has been derived from kinetic theory and applies only to dilute gases, i.e. to real gases in the limit of zero pressure. On the other hand, it does estimate reasonably correctly, and certainly to the order required for a small correction the effect of binary collisions of unlike molecules. The effect of binary collisions between like molecules is fully accounted for by inserting the experimental values of ciscosity μ_{H_2O} and μ_{air} into the equation. Accordingly, we imagine that the effect of increasing the total pressure from 1 atm to P, at constant temperature and composition, is equivalent to so modifying the properties of the molecules that the effect of the binary collisions of like molecules is an increase in the viscosity of the pure components to μ'_{H_2O} and μ'_{air}, and assume entirely heuristically, that the viscosity of the resulting mixture is still represented substantially correctly by the equation

$$\mu'_{mix} = \frac{\mu'_{H_2O}}{1 + \frac{x_{air}}{x_{H_2O}} \cdot \frac{\left[1 + \left(\frac{\mu'_{H_2O}}{\mu'_{air}}\right)^{1/2} \left(\frac{M_{H_2O}}{M_{air}}\right)^{1/4}\right]^2}{2 \cdot 83 \left[1 + \frac{M_{H_2O}}{M_{air}}\right]^{1/2}}} +$$

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* The possibility of an error in mol fraction due to the effect of the total pressure on the partial pressure of the water vapour was considered—investigation showed that this is possible at 25, 50 and 75°C but that the maximum discrepancy from this source is 0·05 per cent. It is believed that the presence of non-equilibrium states is unlikely.

$$\frac{\mu'_{\mathrm{air}}}{1+\frac{x_{\mathrm{H_2O}}}{x_{\mathrm{air}}}\,\frac{\left[1+\left(\frac{\mu'_{\mathrm{air}}}{\mu'_{\mathrm{H_2O}}}\right)^{1/2}\left(\frac{M_{\mathrm{air}}}{M_{\mathrm{H_2O}}}\right)^{1/4}\right]^{2}}{2{\cdot}83\left[1+\frac{M_{\mathrm{air}}}{M_{\mathrm{H_2O}}}\right]^{1/4}}}\,.$$

In reality the change in the viscosity of the pure components $\mu_{\mathrm{H_2O}}$ to $\mu'_{\mathrm{H_2O}}$ and from μ_{air} to μ'_{air} is produced by compression. Hence, the change in the viscosity of the mixture may be written

$$\Delta\mu = \mu'_{\mathrm{mix}} - \mu_{\mathrm{mix}} \approx \frac{\mu'_{\mathrm{H_2O}} - \mu_{\mathrm{H_2O}}}{1+C_1\frac{x_{\mathrm{H_2O}}}{x_{\mathrm{air}}}} + \frac{\mu'_{\mathrm{air}} - \mu_{\mathrm{air}}}{1+C_2\frac{x_{\mathrm{H_2O}}}{x_{\mathrm{air}}}}$$

Here μ'_{mix} is measured at the total pressure P, at temperature T and at composition $x_{\mathrm{H_2O}} + x_{\mathrm{air}} = 1$, whereas μ_{mix} would be measured at the total pressure of 1 atm, and the same temperature and composition as for μ'_{mix}. The values of C_1 and C_2 are easily determined from a knowledge of the molecular weights and viscosities of the pure components.

Table 5. The viscosity of humid air

(*a*) *Series* 1

Temperature t (°C)	Mol Fraction $x_{\mathrm{H_2O}}$	Viscosity μ (micropoise)
25	0·033	183·74
	0·017	184·20
	0·032	183·99
	0·016	184·19
	0·011	184·41
	0·008	184·46
	0·006	184·51
50	0·120	192·47
	0·062	194·74
	0·042	195·39
	0·031	195·75
	0·024	195·91
	0·019	195·93
75	0·371	187·92
	0·265	195·86
	0·193	200·46
	0·132	203·57
	0·097	204·97
	0·075	205·88
	0·055	206·32
	0·387	187·81
	0·317	192·52
150	0·000	239·74
	0·228	222·87
	0·522	196·54
	0·642	178·59
	0·743	162·60

Table 5—continued

(*b*) *Series* 2

Temperature t (°C)	Mol Fraction $x_{\mathrm{H_2O}}$	Viscosity μ (micropoise)
150	0·000	240·71
	0·000	240·57
	0·332	220·68
	0·434	209·31
	0·549	193·37
	0·647	178·76
	0·712	166·93
	0·790	156·66
200	0·000	260·26
	0·311	242·89
	0·402	232·74
	0·492	219·76
	0·797	173·78
	0·000	261·18
	0·697	191·54
	0·000	260·98
	0·701	194·34
	0·000	260·87
	0·632	202·46
	0·000	260·94
	0·605	206·70
	0·000	261·78
250	0·000	281·84
	0·230	272·16
	0·498	243·03
	0·575	231·64
	0·661	217·26
	0·000	281·72
	0·795	192·92
	0·000	281·74
	0·695	212·40
	0·000	281·93
	0·598	229·05
	0·000	281·68
	0·764	196·14
	0·000	282·12
	0·356	261·24
	0·799	192·19

Along the 150, 200 and 250°C isotherms, measurements were performed with a constant mass of air in the instrument. The moisture was introduced by injecting water which evaporated into the air. Thus, once again, the total pressure in the instrument increased, and the use of Dalton's law permitted us to determine the moisture content. The reduction to a nominal pressure of 1 atm was based on the same principle as before.

The appearance of inflection points in the results for the 150, 200 and 250°C isotherms at approximately $x_{H_2O} = 0{\cdot}5$ was sufficiently surprising to make it necessary to confirm that no leakage of air had occurred. The test for leakage was performed by determining, in quick succession, the viscosity of dry air and that of a mixture of high moisture content. In this manner, and in contrast to the previous measurements, the air–vapour mixture did not reside in the instrument for a period of 3 days but only for 2 h. Thus, if any leakage existed, the difference in the amount of air lost would appear as a significant difference in the viscosity measured owing to the very large difference in the time available for the leakage to take place.

The above test was carried out several times but proved completely negative, both procedures yielding substantially identical results.

The results corrected to 1 atm and to nominal temperatures, are seen listed in Table 5: they are presented graphically in Fig. 4. The correction to nominal temperatures was performed using the factor 0·42 micropoise/degC, the largest correction being 1·8 per cent: the largest pressure correction was 1·1 per cent. The viscosity of pure steam was taken from reference 7.

A comparison between the smooth curves

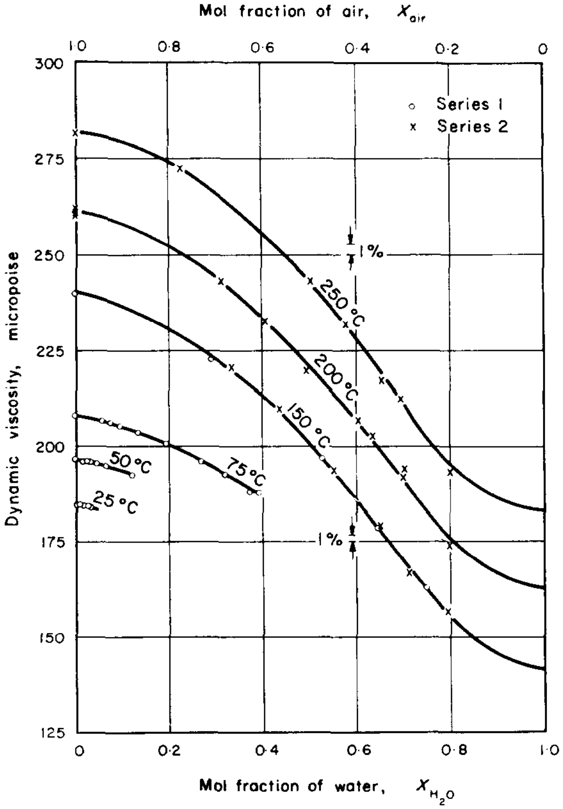

FIG. 4. The viscosity of humid air.

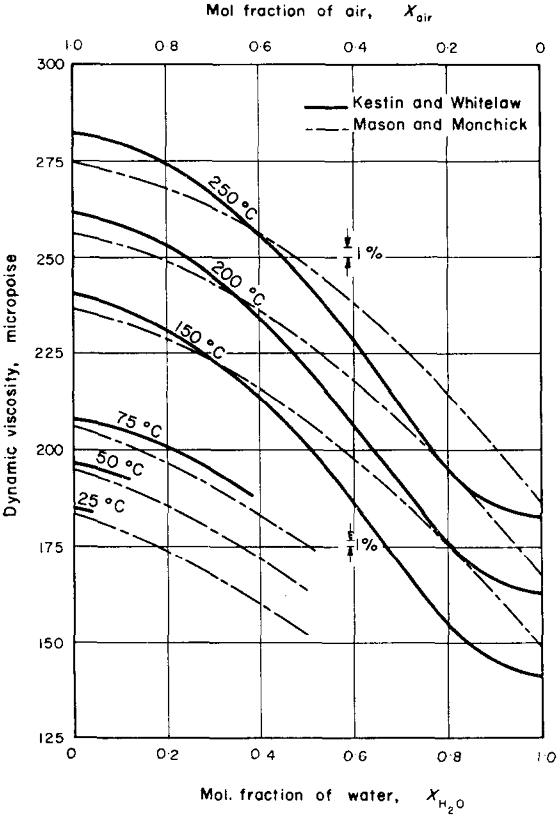

FIG. 5. Comparison between the present determination and the calculations of Mason and Monchick.

shown in Fig. 4 and the theoretical evaluations of E. A. Mason and L. Monchick is given in Fig. 5. It may be seen that there is a maximum discrepancy of approximately 12 per cent at high values of the mol fraction of water and that the theory does not predict the inflection of the curves found by experiment. However, satisfactory agreement is found between theory and experiment at low values of the mol fraction of water, particularly at the lower temperatures, if the value of the pure air component is discounted by considering the ratio $\mu_{\text{mix}}/\mu_{\text{air}}$.

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A shorter version of this paper has been presented at the 1963 International Symposium on Humidity and Moisture and was published in the Proceedings of the Symposium. The present paper recapitulates the previous results and contains results of measurements performed after the Symposium.

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Résumé—L'article présente les résultats de nouvelles mesures de la viscosité de l'air sec le long de sept isothermes dans la gamme 25 à 250°C et 1 à 140 atm, sujette à une incertitude de ±0,4 pour cent. La viscosité de l'air sec pour une densité nulle peut être représentée dans la limite de l'erreur expérimental par une formule d'interpolation basée sur le potentiel 6–12 de Lennard-Jones. Les constantes moléculaires apparentes qui conduisent à une bonne représentation diffèrent considérablement de celles citées dans la littérature. L'effet de la pression peut être pris en compte, au moins approximativement, en ajoutant une fonction en polynôme du 3° degré de la densité à la viscosité pour une densité nulle. Cette fonction, connue comme la viscosité supplémentaire $\Delta\mu$, est indépendante de la temperature, certainement dans la gamme 25 à 100°C.

L'article présente aussi des mesures de la viscosité de mélanges d'air et de vapeur d'eau. Ceux-ci sont restreints à une pression nominale de 1 atm. et convient une gamme de fractions molaires de zéro à 0,03 à 25°C et de zéro à l'unité à 250°C. Les mesures ont été effectuées le long des six isothermes 25, 50, 75, 150, 200 et 250°C. Nous croyons que ce sont les premières mesures de cette espèce.

Zusammenfassung—Es werden die Ergebnisse neuer Viskositätsmessungen von trockener Luft für sieben Isothermen im Bereich von 25–250°C und 1 bis 140 atm mit einer Unsicherheit von ±0,4% angegeben. Die Viskosität von trockener Luft bei der Dichte Null kann innerhalb des Versuchsfehlers dem Kurvenverlauf angepasst werden mit Hilfe einer Interpolationsformel, die auf dem Lennard-Jones 6–12 Potential beruht. Die scheinbaren molekularen Konstanten, die gute Anpassung ermöglichen, weichen erheblich von den in der Literatur angegebenen ab. Der Druckeinfluss kann, zumindest angenähert, dadurch berücksichtigt werden, dass eine kubische Polynomfunktion der Dichte zur Viskosität bei der Dichte Null addiert wird. Diese Funktion wird Überschussviskosität $\Delta\mu$ genannt, sie ist im Bereich von 25–100°C sicher von der Temperatur unabhängig.

In der Arbeit werden auch Viskositätsmessungen von Luft–Wasserdampfgemischen angegeben. Sie beschränken sich auf einen Nenndruck von 1 atm und unfassen Molbrüche von 0–0,03 bei 25°C und 0–1 bei 250°C. Die Messungen wurden für sechs Isothermen 25, 50, 75, 150, 200 und 250°C durchgeführt. Wahrscheinlich sind dies die ersten Messungen dieser Art.

Аннотация—Представлены результаты новых измерений вязкости сухого воздуха вдоль семи изотерм в интервале температур от 25 до 250°C и давлений от 1 до 140 атм при погрешности ±4%. Вязкость сухого воздуха при нулевой плотности можно определить с точностью до погрешности измерения по интерполяционной формуле, основанной на потенциале Леннарда-Джоунса 6–12. Кажущиеся молекулярные константы, ведущие к хорошему соответствию, значительно отличаются от приведенных в литературе. Влияние давления можно учесть, по крайней мере приближенно, добавлением зависимости $\rho = f(\mu_0)$ в виде полинома третьей степени. Эта функция, известная как избыточная вязкость $\Delta\mu$, не зависит от температуры, особенно в интервале температур от 25 до 100°C.

В статье представлены также результаты измерений вязкости смесей воздуха и водяного пара для случая номинального давления, равного 1 атм, и диапазона молярных концентраций 0–0,03 при 25°C и 0–1 при 250°C. Выполнены измерения вдоль шести изотерм 25, 50, 75, 150, 200 и 250°C. Авторы полагают, что такие замеры выполнены впервые.